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PROPAGATION OF A TWO-DIMENSIONAL PLASTIC WAVE IN A NONLINEARLY COMPRESSED HALF-PLANE

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We shall consider a two-dimensional stationary problem of the propagation of a shock wave in a nonelastic ideal medium filling a half-space, when a moving load acts on its boundary. The solution of the problem is constructed by the method of characteristics for the case where the velocity of motion of the load exceeds the velocity of propagation of the shock wave in the medium whose compressibility is nonlinear and irreversible (Fig. 1), while in [1] the case of a linear relation between p and ε is investigated analytically. At the same time the surface of the medium where the pressure is applied is assumed, just as in [2, 3], only little deformed, and therefore it is assumed that the pressure is applied to a horizontal nondeformed surface (Fig. 2).

The scheme proposed provides us with a possibility of carrying out the calculation of the parameters of the medium (in particular, of the ground) that is being modeled by a generalized plastic gas [2] or an ideal liquid [4], and also in the case of wave propagation in reservoirs with a screen [3], and so forth.

The results of the numerical calculation are represented in the form of curves of the variation of the pressure and the velocity of the medium in the region of perturbation along the wave front.

Let a monotonically decreasing normal pressure move along the surface of a half-space at a velocity D (see Fig. 2). Then in the half-space a shock wave with a curvilinear front Σ will propagate at a velocity a, the value of which is not known in advance and is determined in the solution process of the problem.

We shall assume that the medium on the front Σ is instantaneously loaded, while behind the front (in the perturbed region) there occurs unloading which is assumed to be linear. In this case on the front Σ from the condition of conservation of the mass and the impulse we obtain

$$\rho_0 a = \rho^*(a - v_n^*),\ \rho_0 a v_n^* = p^*,\ v_\tau^* = 0, \tag{1}$$

while the equation of state of the medium is represented in the form of a polynomial

$$p^* = \alpha_1 \varepsilon^* + \alpha_2 \varepsilon^{*2}. \tag{2}$$

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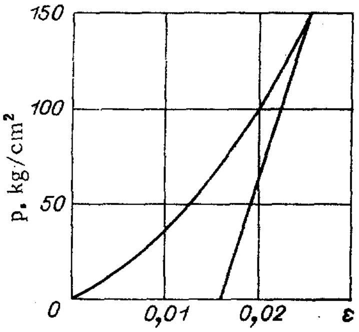

Fig. 1

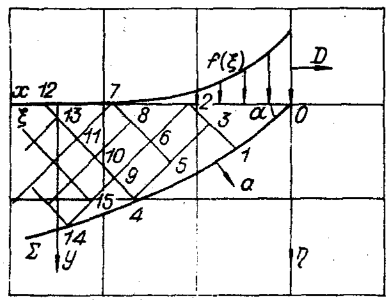

Fig. 2

In the region of unloading (i.e., within the angle α) we have the equations

$$\begin{cases} \frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \ \frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \\ \frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0; \end{cases} \tag{3}$$

$$p = p^* + E_p(\varepsilon - \varepsilon^*). \tag{4}$$

The boundary condition for $y = 0$, $Dt + x \geqslant 0$ has the form

$$p = f(Dt + x), \tag{5}$$

where $f(\xi)$ is a known function; v_τ^*, v_n^* are the tangential and normal components of the mass velocity V of the medium relative to the wave front Σ; u and v are, respectively, the projections of the velocity onto the x and y axes; α_1, α_2, E_p are constant quantities; ε, ρ are the volumetric strain and the density of the medium.

With the aim of constructing the solution of the problem by the method of characteristics, we go over to the moving coordinate system $\xi = Dt + x$, $\eta = y$, and introduce the velocity potential

$$u = \partial\varphi/\partial x, \ v = \partial\varphi/\partial y.$$

Then from (3) we have the equations of characteristics and the characteristic relations

$$\eta'_{1,2} = \frac{d\eta}{d\xi} = \pm\frac{1}{\sqrt{D^2/c_p^2 - 1}}, \ d\varphi_\xi + \eta'_{2,1} d\varphi_\eta = 0, \tag{6}$$

as well as the Bernoulli expression in the form

$$dp = \rho D d\varphi_\xi \left(c_p^2 = \frac{E_p}{\rho}\right), \tag{7}$$

where $D > c_p$.

Bearing in mind the fact that Eq. (4) can be used for determining the volumetric strain ε, the system of equations (6), (7), with (1), (5) and the equations of state (2), (4) taken into account, is solved on a computer for the case where the given load varies along ξ according to the law

$$f(\xi) = p_0(1 - \xi/n_0); \tag{8}$$

$$f(\xi) = p_0 \exp(-\xi/m_0), \tag{9}$$

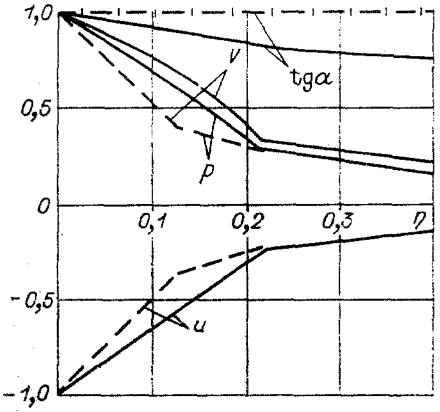

Fig. 3

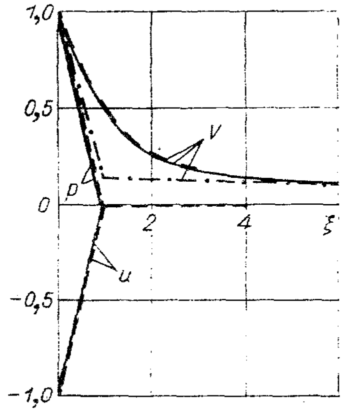

Fig. 4

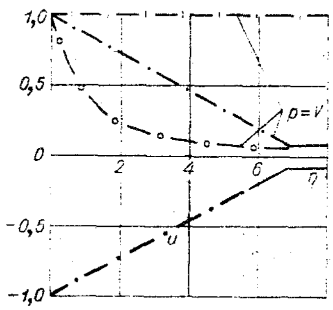

Fig. 5

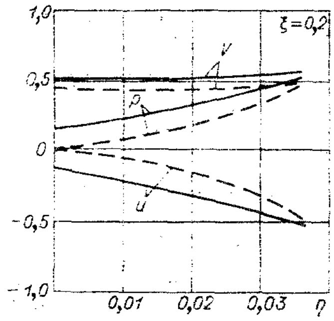

Fig. 6

where p_0, n_0, m_0 are constant quantities having the dimensionality of force and length, respectively.

The numerical calculation scheme presented in Fig. 2 consists of the following. At first from Eqs. (1), (2), with (5) taken into account, we determine all parameters of the point 0, including the angle of inclination of the shock wave α. Then from the point 0 (see Fig. 2) we draw a segment 01 at an angle α, and the parameters of the medium at the point 0 are transferred to the point 1. Such an approximation is more accurate for smaller lengths of the segment 01. From the point 1 we draw the characteristic of the second family 1, 3, 2 that intersects the boundary of the half-space at the point 2. From Eqs. (4)-(7), represented in terms of finite differences, we determine the parameters of the point 2. At the middle of the segment 1, 3, 2 we choose a point 3. Its parameters are found as the arithmetic means of the parameters of the medium at the points 1 and 2. Then from the point 3 we draw the characteristic of the first family as far as its intersection with the continuation of the segment 01 to the point 4 and so forth.

On the basis of the scheme presented above the problem is solved numerically by means of a computer for $m_0 = 0.1$, $n_0 = 0.15, 1.0$; certain results of the calculation in the form of graphs of p, u, v and tan α are represented in Figs. 3-6 in dimensionless form in relation to their maximum values, while the variables ξ and η are referred to a unit of length. All curves calculated for (8) are shown in Figs. 3-5, where the solid lines refer to a nonlinear, and dashed lines refer to a linear medium; the dashed-dot lines refer to the case where α = 45°, the dashed lines with circles refer to the ray approximation [5], while the dashed lines with vertical line segments represent the acoustics. In Fig. 6 the solid lines refer to the case (8), while the dashed lines refer to the case (9); the variation of the parameters of the medium along the front are presented in Figs. 3 and 5, while the variation along the free surface is presented in Fig. 4.

These curves allow us to study the influence of the properties of the medium and the load on the shock wave processes, the kinematic parameters, and the pressure in the half-space. It is established that in contrast to the linearly compressed medium [1], nonlinear compressibility of the material of the half-space leads to an increase in the values of pressure and mass velocity along the front (Fig. 3), to widening of the region of perturbation, and to a decrease in the vertical component of the velocity on the free (boundary) surface (Fig. 4).

If the shock wave forms an angle $\alpha < 45°$ with the 0ξ axis, then the variation of the vertical component along 0ξ becomes smooth and less shallow than for $\alpha = 45°$.

In the last case the comparison of the results of the numerical calculation and the analytical solution [1] carried out shows that all parameters of the medium on the wave front mutually agree with an accuracy of 0.1%, while in the case of the ray approximation [5] their values are somewhat reduced. We further discover that the character of variation of the load profile along the boundary surface substantially alters the pressure distribution both in depth and along the half-space. On the basis of the analysis of the results of the calculation we note that p, u, v in the region of aftereffect of the moving load $\xi > 1$, dependent on the depth, vary according to a nonlinear law (in contrast to the region of application of the load $\xi \leqslant 1$).

From Fig. 6 we see that for $\xi = 0.2$ an exponential load in comparison with a load of finite length leads to an increase in the values of p, u, v at all points of the half-space, which was to be expected, since the value of the applied load (9) on the free surface is greater than (8).

The given scheme allows us to calculate the parameters of a nonlinearly compressed half-plane also in the case of nonlinear unloading of the medium.

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APPROXIMATE EQUATIONS OF DYNAMICS OF AN ELASTIC LAYER

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With the investigation of wave phenomena in an elastic layer, wide use has been made of approximate theories based on representation of the displacements in the form of series with respect to the middle surface [1-3]. Expansion in series in terms of Legendre polynomials is one method for the representation of the sought solution of the theory of elasticity, and has advantages over the remaining methods [1]. Retaining one number of terms or another for the coefficients of the series, different variants of the equations of the dynamics of plates can be derived. The equations of Bernoulli–Euler and Timoshenko have been the most completely investigated. In addition, for the description of processes taking place in a layer, in recent years different variants of the refined equations have been brought in [2, 4, 5].

The interest in the vibrations of plates, and the derivation of more exact equations, is connected partially with the fact that a transition from the equations of the theory of elasticity to approximate equations leads to errors in the description of non-steady-state processes. Thus, due to the approximate manner of taking account of the distribution of the displacements over the thickness of the layer, no account is taken of surface Rayleigh waves or of the fronts of waves reflected repeatedly from the surfaces of the layer; i.e., in the derivation of the equations of plates, high frequencies are ignored. These rapidly varying parts of the solution of the theory of elasticity are determined in the expanded terms of the dis-

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